01204528 Queueing Theory and Applications in Networks

Homework Assignment (7 Problems)

Due date: ศุกร์ที่ 30 ก.ค. 2553 ก่อนเวลา 11:00 น.

(ส่งที่ช่องตู้ไม้ซื่อ อ.อนันต์ ผลเพิ่ม หน้าห้องธุรการภาควิชา ชั้น 4)

1. The cumulative distribution function of random variable M is

$$F_M = \begin{cases} 0 & m < -5 \\ \dfrac{(m+3)^2}{81} & -5 \le m < 4 \\ 1 & m \ge 4 \end{cases}$$

   a. Find the E[M]
   b. Find the Var[M]

2. Long-distance calling plan A offers rate service at 3 Baht per minute. Calling plan B changes 25 Baht for every call under 20 minutes; for calls over 20 minutes, the charge is 25 Baht for the first 20 minutes plus 3 Baht for every additional minutes (Note that these plans measure your call duration exactly, without rounding to the next minutes or even second.) If your long-distance calls have exponential distribution with expected value β minutes, which plan offers a lower expected cost per call?

3. For 85% of lectures, ProfessorX arrives on time and start lecturing with delay D = 0. When Professor X is late, the starting time delay D is uniformly distributed between 0 and 250 seconds. Find the CDF and PDF of D.

4. S and T are random variable with the joint PDF

$$f_{S,T}(s,t) = \begin{cases} 3 & s + t \le 1, s \ge 0, t \ge 0 \\ 0 & Otherwise \end{cases}$$

   a. Find the marginal PDF $f_S(s)$
   b. Find the marginal PDF $f_T(t)$

5. In a factory produces resistors for the value of 10 kΩ, the resistance of each resister is a random variable R uniformly distributed between 9,965 and 10,300 Ω. The resistance for each R is independent from each others. The factory has an order for one-part-in 10^4 resistors with the resistance value of 9,995 and 10,005 Ω. An inspector person takes one R per second from the production line and measure the exact resistance value. (Assume that the test take one second.) The randon variable T_X seconds is the elapsed time at which the inspector person finds x acceptable resistor.
   a. What is the probability (p) that any single resitance has one-part-in 10^4 accuracy?

b. What is the $E[T_1]$ seconds, the epected time for the inspector to find the first one-part-in 10^4 resistance value.
c. What is the probability that the inspector will find the first one-part-in 10^4 resistor in exactly 30 seconds?
d. What is E[T4], the expected time of finding the forth one-part-in 10^4resistor.

6. For a search engine company, queries presented to a serach engine with a Poisson process of rate λ = 5 queries per minute. An experiment consists of monitoring the search engine for w minutes and recording N(w), the number of queries presented. The answer to each of the following questions can be experessed in terms of the PMF $P_{N(w)}(k) = P[N(w)= k]$.
    a. Find the probability of no queries in a one minute interval
    b. Find the probability of exactly 5 queries arriving in one-minute interval
    c. Find the probability of exactly 4 queries arriving in a two-minute interval

7. Customers arrive at a KU book store as a Poisson process of rate 40 customers per hour. Upon arriving, each customer must flip a coin, and only those customers who flip heads actually enter the book store. Let N(t) denote the process of customers entering the book store. Find the PMF of N, the number of customers who arrive between 8:30 AM to 11:30 AM.