Lecture # 12
M/G/1 Busy Period

ดร.อนันต์ ผลเพิ่ม
Anan Phonphoem, Ph.D.

anan@cpe.ku.ac.th
http://www.cpe.ku.ac.th/~anan
Computer Engineering Department
Kasetsart University, Bangkok, Thailand

Outline

- Busy period and its duration
- M/G/1 with Vacations

Busy and Idle period

- To derive the distribution for the M/G/1 queue
  - the length of the Idle period
  - the length of the busy period

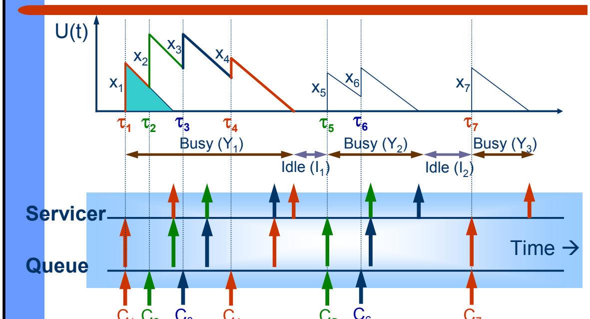
Busy and Idle period

- C_n = the n^{th} customer to enter the system
- τ_n = arrival time of C_n
- $t_n = \tau_n - \tau_{n-1}$ = interarrival time between C_{n-1} and C_n
- x_n = Service time of C_n

Busy and Idle period

- U(t) = Unfinished work in the system
  = Virtual waiting time at time t
- Y_n = Busy period
- I_n = Idle period

The Unfinished Work (FCFS)



M/G/1 (FCFS)

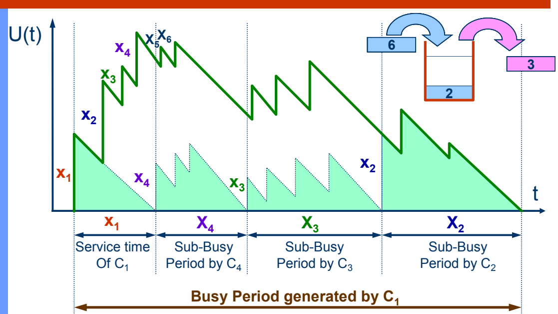
- $A(t) = P[t_n \le t] = 1 - e^{-\lambda t} \quad t \ge 0$
- $B(x) = P[x_n \le x]$
- A(t) and B(x) are independent on n
- F(y) = Idle period distribution
  $= P[I_n \le y]$
- G(y) = Busy period distribution
  $= P[Y_n \le y]$

M/G/1 (FCFS)

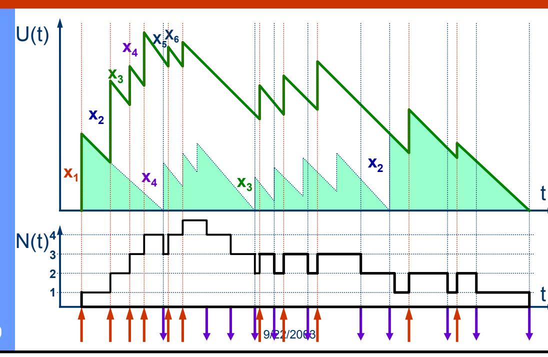
- For Idle period F(y)
  - After busy period ➔ start the idle period
  - A new idle period will stop immediately when the new customer arrives
  - Therefore, from the memoryless distribution

  $F(y) = 1 - e^{-\lambda y} \quad y \ge 0$

The Unfinished Work (LCFS)



Number of Users (LCFS)



M/G/1 (LCFS)

- Each sub-busy period behaves statistically the same as the major busy period
- The duration of busy period Y

  $Y = x_1 + X_{v+1} + X_{v+2} + \ldots + X_3 + X_2$

  X_v = sub-busy period

  v = an RV = # of customer arrives during C_1 service interval

M/G/1 (LCFS)

- For Busy period G(y)

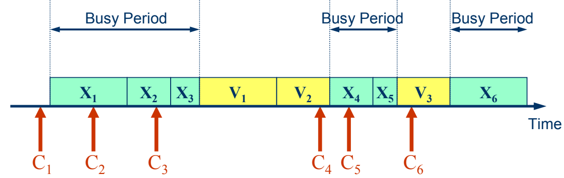
  $G(y) = P[Y_n \le y] \quad y \ge 0$
- Transform of M/G/1 busy-period distribution

  $G^*(s) = B^*[s + \lambda - \lambda G^*(s)]$

M/G/1 with Vacations

- At the end of busy period
  - The server goes on "vacation"
  - The vacation period = random interval of time
  - A new arrive during vacation has to wait until the end of vacation period
  - If the system is idle after vacation, a new vacation starts right away

M/G/1 with Vacations



- V_n = Vacation period with $\overline{V}$ and $\overline{V^2}$
  = IID random variable and independent of customer interarrival and service time
- X_n = Service period

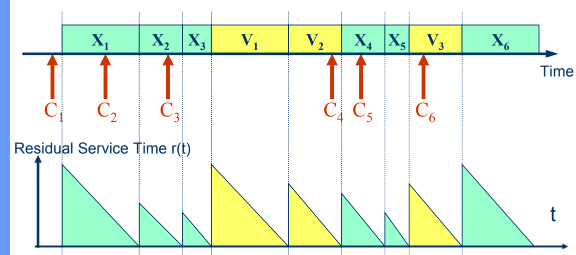
M/G/1 with Vacations

- A new customer is Possion arrival and service time is general distribution
- The waiting time for customer is W

$$W = \frac{R}{1-\rho}$$

- R = Residual Time

M/G/1 with Vacations



M/G/1 with Vacations

$$R = \frac{1}{t}\int_0^t r(\tau)\,d\tau = \frac{1}{t}\sum_{i=1}^{M(t)}\frac{1}{2}X_i^2 + \frac{1}{t}\sum_{i=1}^{L(t)}\frac{1}{2}V_i^2$$

$$= \frac{M(t)}{t}\,\frac{\sum_{i=1}^{M(t)}\frac{1}{2}X_i^2}{M(t)} + \frac{L(t)}{t}\,\frac{\sum_{i=1}^{L(t)}\frac{1}{2}X_i^2}{L(t)}$$

M/G/1 with Vacations

$t \rightarrow \infty \quad M(t)/t = \lambda$ and $L(t)/t = (1-\rho)/\overline{V}$

$$R = \frac{1}{2}\lambda\overline{X^2} + \frac{(1-\rho)\overline{V^2}}{2\,\overline{V}}$$

$$W = \frac{\lambda\overline{X^2}}{2(1-\rho)} + \frac{\overline{V^2}}{2\,\overline{V}}$$