

Error Detection and Correction

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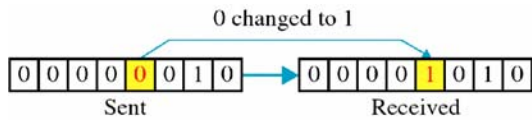
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Errors

Single-Bit

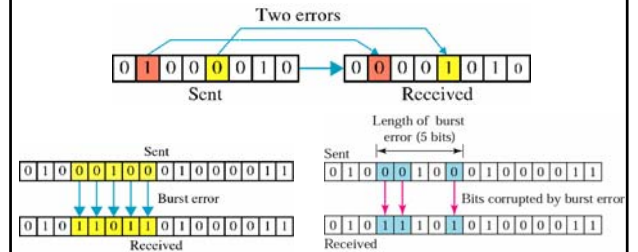
Burst (Multiple-bit)

Single-bit error



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Burst error (Multiple-bit error)



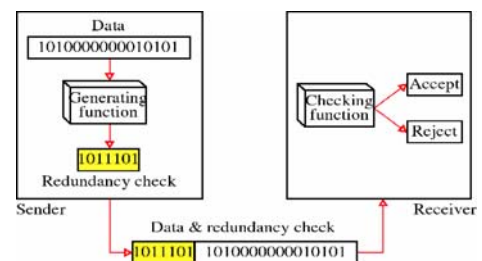
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How do we know that data is corrupted (errors occur)?

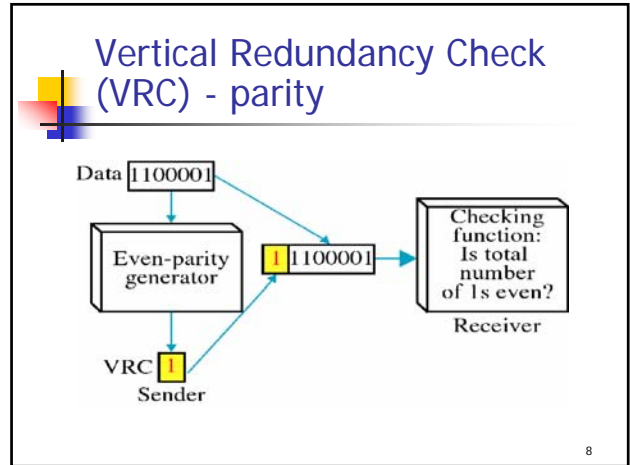
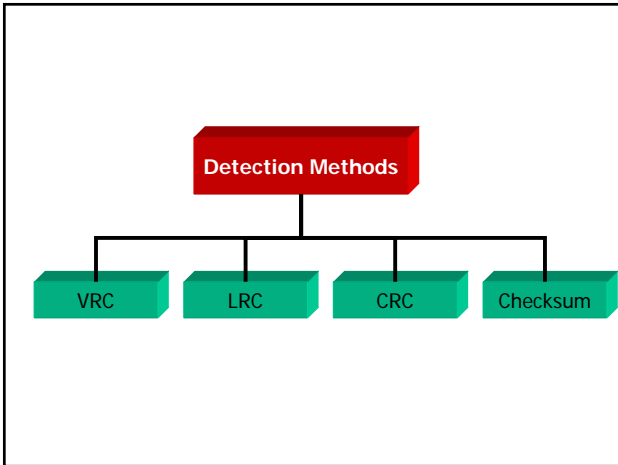
- How to detect the error?
- Does not know at the signal level
- Comparison...
- Redundancy
 - 2 copies of everything (slow, high overhead)
 - Extra info.

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Redundancy



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Example

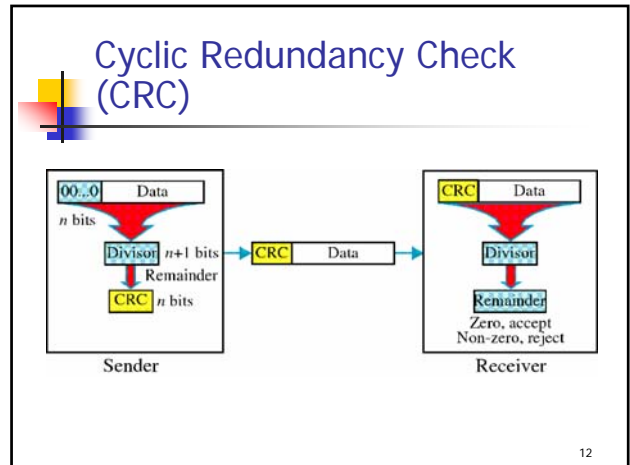
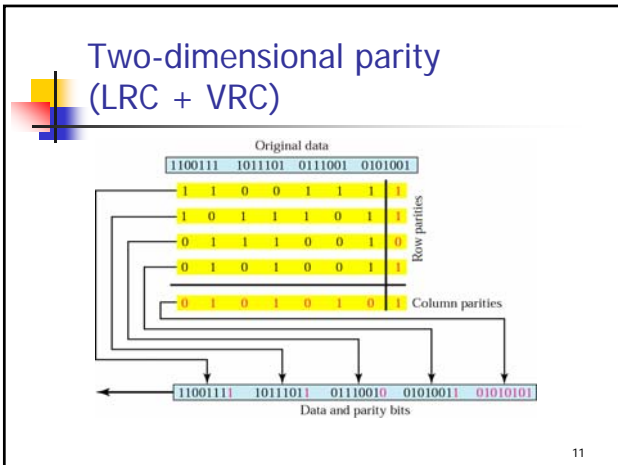
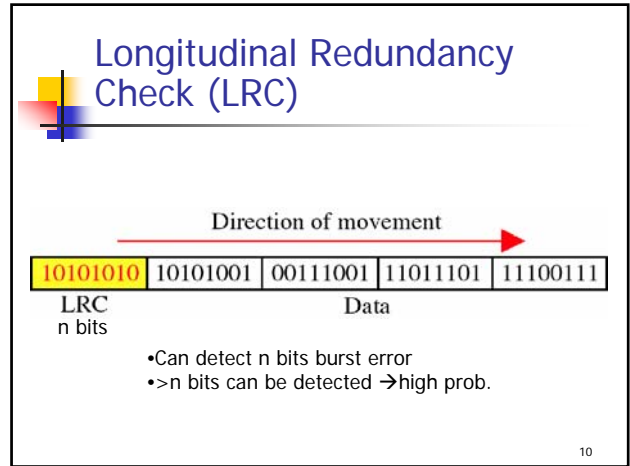
Suppose the sender wants to send the word *world*. In ASCII the five characters are coded as

1110111 1101111 1110010 1101100 1100100

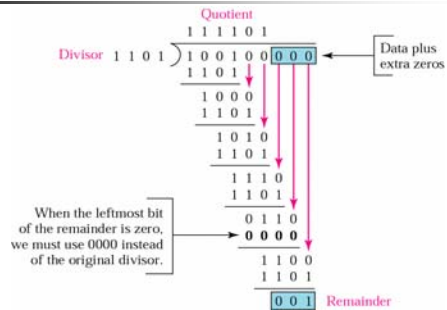
w o r l d

The following shows the actual bits sent

11101110 11011110 11100100 11011000 11001001

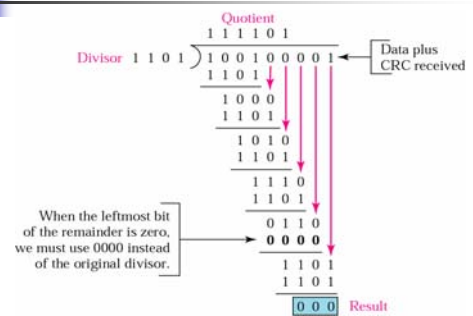


Binary Division in a CRC generator



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Binary Division in a CRC checker



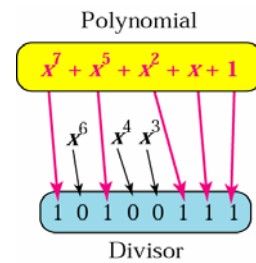
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Polynomial

$$x^7 + x^5 + x^2 + x + 1$$

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Polynomial and Divisor



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Choosing the polynomial

2 properties

- Should not be divisible by x
- Should be divisible by $(x+1)$

Example

- Cannot choose x (binary 10) or $x^2 + x$ (binary 110)
 - Because both are divisible by x .
- Can choose $x + 1$ (binary 11)
 - Because it is not divisible by x , but is divisible by $x + 1$.
- Also choose $x^2 + 1$ (binary 101)
 - Because it is divisible by $x + 1$ (binary division).

Standard Polynomials

Name	Polynomial	Application
CRC-8	$x^8 + x^2 + x + 1$	ATM header
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^2 + 1$	ATM AAL
ITU-16	$x^{16} + x^{12} + x^5 + 1$	HDLC
ITU-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$	LANs

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CRC

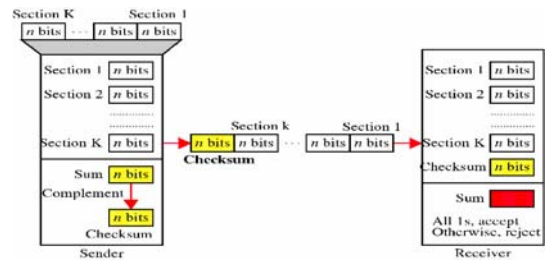
- Very effective detection method
- Can detect all burst of length $<$ degree of polynomial
- Detect other errors \rightarrow high prob.

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Checksum

Notes:

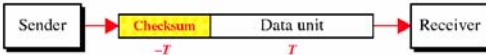
- Segment
- Add using one's complement
- Sum is complemented



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Data Unit and Checksum

The receiver adds the data unit and the checksum field. If the result is all 1s, the data unit is accepted; otherwise it is discarded.



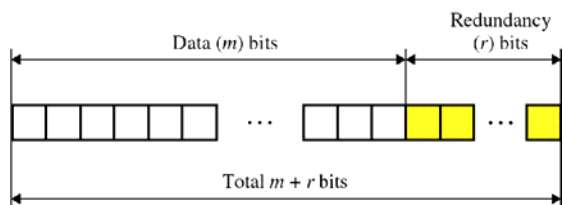
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When the error is detected:

- How can we correct them?
 - Discarded and resent
 - Automatically correct without resent
- In theory, any error binary code can be corrected
 - But adding complexity
 - Redundancy
 - Limited up to 3 errors because of overhead

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Error Correction



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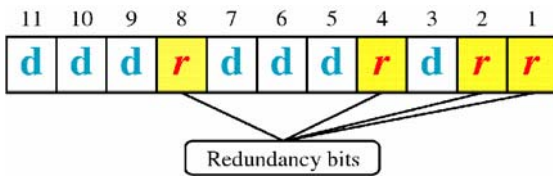
Number of redundancy bit

- Total bit = $m+r$
- r must be able to indicate all positions including no error $\rightarrow m+r+1$
- r bit can indicate 2^r states
- So $2^r \geq m+r+1$
- For ASCII (7bits) $\rightarrow m = 7$
 - $2^4 \geq 7+4+1$

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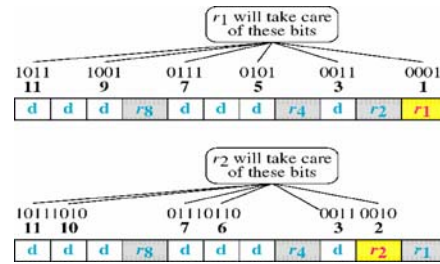
Hamming Code

- Example: 7-bit ASCII needed $r = 4$



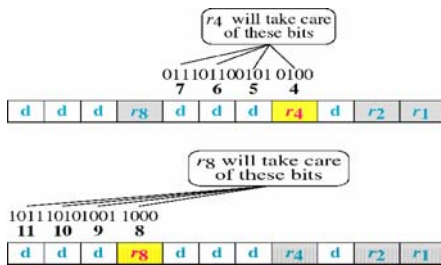
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Hamming Code



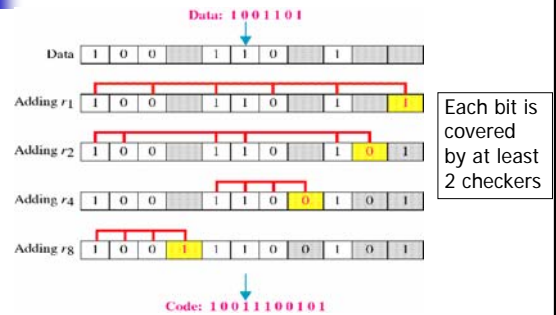
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Hamming Code



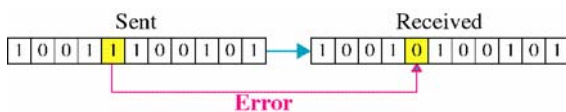
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Example of Hamming Code



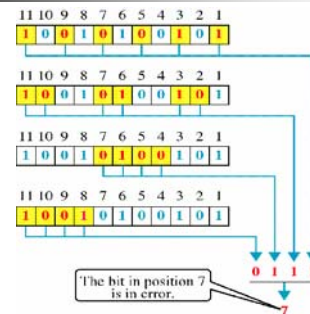
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Single-bit error



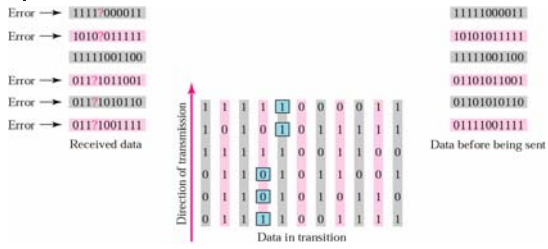
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Error Detection



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Burst Error Correction



Rearrange the data to become one bit error