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Probability and Random Processes
for Computer Engineering
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Intro to Renewal Processes, Markov Chains, and Queueing Theory

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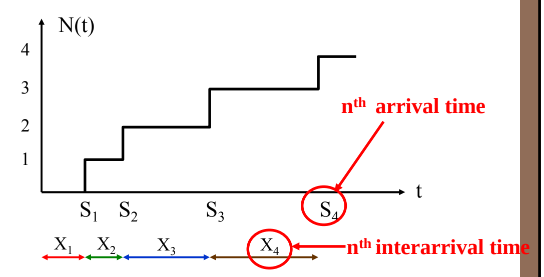
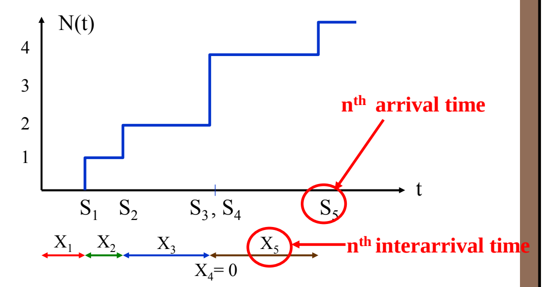
Counting Process



Counting Process

- N(t) is stochastic process
- Non-decreasing and integer
- $t \geq 0$; $N(0) = 0$
- One arrival at a time
- How about more than one arrival at a time?

Counting Process (>1 arrival)



Renewal Process

- **Definition:**
  A renewal process N(t) is a counting process for which the interarrival times $X_1, X_2, X_3, \ldots$ are an iid random sequence

Renewal Process

- N(t) is the $F_X(x)$
- Interarrival = iid
- After arrival, time until next arrival has the same distribution

The process restarts (renewal) when an arrival occurs

Renewal Process

- After n arrivals, joint CDF of the next k arrivals is

$$F_{X_{n+1},\dots X_{n+k}}(x_1,\dots,x_k) = F_X(x_1)F_X(x_2)\dots F_X(x_k)$$
$$= F_{X_1,\dots X_k}(x_1,\dots,x_k)$$

Discrete Time Markov Chains

Definition: Discrete Time Markov Chains $\{X_n | n = 0,1,2,\dots\}$ **,** $\mathbf{X_{n+1}}$ **depends only on** $\mathbf{X_n}$ **through the transition probability**

$$P[X_{n+1}= j \mid X_n= i , X_{n-1}= i_{n-1} ,\dots, X_0=i_0]$$
$$= P[X_{n+1}= j \mid X_n= i]$$
$$= P_{ij}$$

Discrete Time Markov Chains

- X_n summarizes all the past history
- X_n = state of the system @ time n

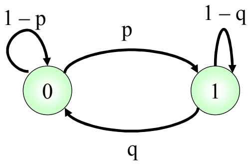
Theorem: Transition Probability P_{ij} of a Markov Chains

$$P_{ij} \geq 0 \qquad \sum_{j=0}^{\infty} P_{ij} = 1$$

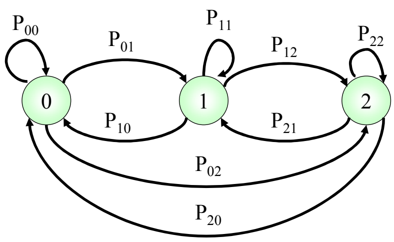
Discrete Time Markov Chains

$$P = \begin{pmatrix} P_{00} & P_{01} & \dots & P_{0K} \\ P_{10} & P_{11} & \dots & P_{1K} \\ \dots & \dots & \dots & \dots \\ P_{K0} & P_{K1} & \dots & P_{KK} \end{pmatrix}$$

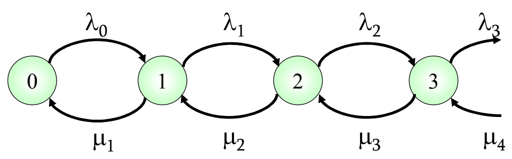
Two-state Markov Chain



Three-state Markov Chain



Birth-Death Process



The Queue and I

- Real life situation
  - Wait for buying lunch
  - Wait for taking a ride in Disney World
  - Wait for withdraw money from ATM
  - Wait for a green light
  - Wait for Bug 1113 to pick up our call
  - Etc.

Who like to wait?

- Customer does not
- Entrepreneur does not like it either
  - Cost more money
  - Cost more space for waiting
  - Customers loss
  - Unhappy customers

So, why waiting?

- Demand > Service available
- Why service is not enough?
  - Not economic
  - No space
  - Unpredictable arrival

Still Waiting …

- Interesting questions for customers?
  - How long do I need to wait?
  - How many people in the line?
  - When should I come to get serve faster?

Still Waiting …

- Interesting questions for service provider?
  - How big is the waiting area?
  - How many customers leave?
  - Should we add some more tellers?
  - Should the system form 1 or 3 queues?
  - Should the system provide a fast lane?

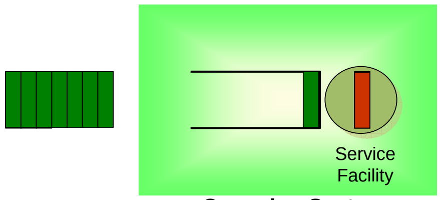
Here comes …Queueing Theory

- Describe the queue phenomena
  - Waiting and serving
- Model the system mathematically
- Try to answer those questions

Queueing System

- Arriving for service
- Waiting for service
- Getting serve
- Leaving the system

General queueing system



Queueing System

Queueing process characteristics

- Arrival pattern
- Service pattern
- Queue discipline
- System capacity
- Number of service channels
- Number of service stages

Arrival pattern

- Stochastic
  - Probability distribution
  - Single or batch arrival
- Behavior of customer
  - Patient customer
    - Wait forever
  - Impatient customer
    - Wait for a period and decide to leave
    - See the long line and decide not to join
    - Change the waiting line

Arrival pattern

- Is it time dependent?
  - Stationary arrival pattern (time independent – probability distribution)
  - Nonstationary arrival pattern

Service pattern

- Distribution for service time
- Single or batch (parallel machine) service
- Service process depends on number of customers waiting (state dependent)
- Very fast service → still have a line?
  - Depends also on the arrival
  - May assume mutually independent

Queue discipline

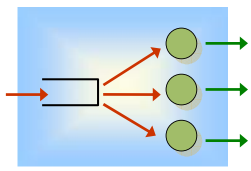
- Manner of customers to get serve
- First come, first serve
- Last come, first serve
- Random serve
- Priority serve
  - Preemptive
  - Nonpreemptive

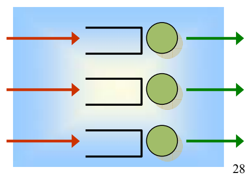
System capacity

- Finite capacity
  - Maximum system size
- Infinite capacity

Number of service channels

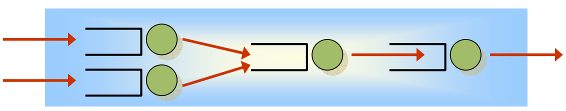
- Multiserver queueing system
  - Single line service
  - Multiple line service





Stages of service

- Single stage
- Multiple stages
  - Without feedback (Entrance Exam)
  - With feedback (Manufacturing)



Queueing Notation

- Kendall’s notation (1953)

A / B / X / Y / Z

A : Interarrival-time distribution
B : Service time distribution
X : # of parallel service channels
Y : System capacity
Z : Queue discipline

Queueing Notation A/B/X/Y/Z

Characteristics	Symbol	Explanation
A & **B** (Interarrival / Service Time)	M D E_k G	Exponential (Memory less) Deterministic Erlang General
X (# Servers)	1,2,…,∞	
Y (Capacity)	1,2,…,∞	
Z (Q discipline)	FCFS, PR	

31

Queueing Notation A/B/X/Y/Z

- M/M/3/∞/FCFS
  - Exponential interarrival time
  - Exponential service time
  - 3 parallel servers
  - Unlimited space
  - First-come first-serve queue discipline

32

Queueing Notation A/B/X/Y/Z

- M/D/1
  - Exponential interarrival time
  - Deterministic service time
  - 1 server
  - (default) Unlimited space
  - (default) FCFS queue discipline

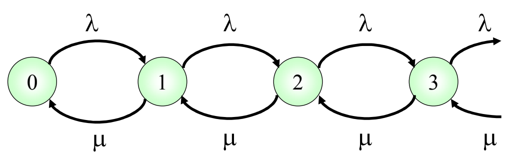
33

Queueing Notation A/B/X/Y/Z

- M/M/1
- M/M/c/k
- M/M/∞
- E_k/M/1
- M/G/1
- G/M/m
- G/G/1

34

M/M/1



Arrival = Exponential
Service = Exponential

35