

204312
**Probability and Random Processes
for Computer Engineering**

1st semester 2004 (June – September)

1

Instructor Information

Instructor: Asst. Prof. Dr. Anan Phonphoem
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2

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3

Course Information

Lecture:
Tu Th 10:30 – 12:00 Room 204

Class URL:
<http://www.cpe.ku.ac.th/~anan/204312.htm>

Prerequisite:
417168 Engineering Mathematics II

Course Description:
Introduction to Probability; Discrete Random Variables;
Continuous Random Variables; Renewal Processes and
Markov Chains

4

Textbook

**“Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer
Engineers,”**

Roy D. Yates and David J. Goodman, John Wiley &
Sons, Inc., 1999, ISBN 0-471-17837-3

5

Supplements

**“Probability and Statistics with Reliability, Queuing and
Computer Science Applications,”** Trivedi, John Wiley &
Sons, Inc., 2002, ISBN 0-471-33341-7

**“Probability, Random Variables, and Stochastic
Processes,”** Athanasios Papoulis, McGraw-Hill, 3rd Edition,
ISBN 0-07-048477-5

**“Probability, Random Variables, and Random Signal
Principles,”** Peyton Z. Peebles, Jr., McGraw-Hill, Second
Edition, ISBN 0-07-100234-0

6

Exam Dates

Midterm Exam:

Mon July 26, 2004, 4-7 PM

Final Exam:

Wed September 22, 2004, 1-4 PM

7

Grading Policy

Quiz: 16 % (2 times – 8% each)

Midterm Exam: 35 %

Final Exam: 35 %

Assignment & Hw: 14 %

Attendance and Class participation:

May be considered for grade adjustment

(eg. B → B+)

8

Notes

- Minimum Score to pass the course is 50 %
- Any form of cheating will receive an F Grade (for all involved parties)
- Copy homework or assignment will be considered as cheating
- Late assignment and homework will not be graded
- No Makeup Quiz

9

Important Dates

Date	Description
June 1	Class Start
June 22	Quiz 1
June 24	Teacher's Day
June 29, July 1, 6	No Class(Traveling Abroad)
July 22	Graduation Ceremony
July	Makeup Class (3 Hrs.)
July	Makeup Class (3 Hrs.)
July 26	Midterm Exam
Aug 12	Holiday
Aug 24	Quiz 2
Sep 16	Class End
Sep 22	Final Exam

10

Lecture #1: Introduction to Probability and Set Theory

11

Probability (ความน่าจะเป็น)

เรานึกถึงอะไร เมื่อพูดถึงความน่าจะเป็น ?

– Physical Property

คช. ทอยลูกเต๋า โยนเหรียญ ลีดเตอรวัลี่

– Knowledge

คช. Web page ที่โดน hack

12

Probability (ความน่าจะเป็น)

- Situation cannot exactly replicate
 - การผลิต IC
- But not chaotic
 - ถ้าทดลองหลายๆ ครั้ง น่าจะมี pattern
 - การโยนเหรียญ 100 ครั้ง
- ค่าของความน่าจะเป็น
 - มีค่า $[0,1]$

13

คำสำคัญ

- **Definition**
 - สร้างตรรกของความน่าจะเป็น
- **Axiom**
 - ความจริงที่ไม่ต้องการการพิสูจน์ (3 ข้อ)
- **Theorem**
 - สิ่งที่ได้มาจาก definition และ axiom
 - ต้องพิสูจน์ได้ด้วย definition, axiom, และ Theorem อื่น

14

Set Theory

- Set เป็นคณิตศาสตร์พื้นฐานของ probability
- คำจำกัดความ
 - Set:** กลุ่มของสิ่งที่สนใจ
 - Element:** สมาชิกที่ประกอบกันเป็น Set
- Set และ Element
 - $a \in A$ **a** is an element of set **A** (set inclusion)
 - $b \notin B$ **b** is not an element of set **B**

15

การกำหนด Set

$T = \{\text{Monitor, Keyboard, Mouse}\}$

$R = \{2x^2+1 \mid x=1,2,3,\dots\}$



{ทางซ้ายเป็นสมาชิกของSet | ตามตัวเลข-เงื่อนไขทางขวา }

16

Set Theory

Subset

$A \subset B$ set **A** is a subset of set **B**

$R = \{2x^2+1 \mid x=1,2,3,\dots\}$

$I = \{\text{Integer Number}\}$

$R \subset I$

Set Equality

$A = B$

$\rightarrow A = B$ if and only if (iff) $A \subset B$ and $B \subset A$

17

Set Theory

Universal Set "S"

- Set ของทุกสิ่งที่เป็นไปได้
- By defintion: ทุก set เป็น subset ของ S

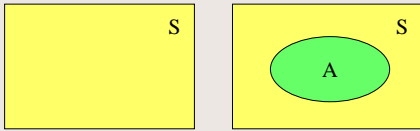
Null Set "∅"

- เซตว่าง เป็นเซตที่ไม่มีสมาชิก
- By defintion: $\emptyset \subset A$

18

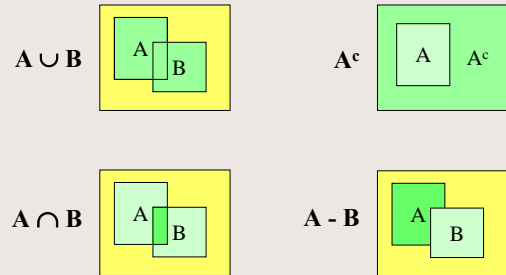
Venn Diagram

- แสดงความสัมพันธ์ของ Set



19

Set Operation



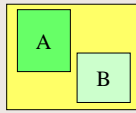
20

Important Set Properties

1. Mutually Exclusive

$$A_i \cap A_j = \emptyset \quad \text{for } i \neq j$$

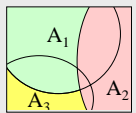
$$A \cap B = \emptyset \rightarrow \text{Disjoint}$$



2. Collectively Exhaustive

$$A_1 \cup A_2 \cup \dots \cup A_n = S$$

$$\bigcup_{i=1}^n A_i = S$$



21

Theorem: De Morgan's Law

$$(A \cup B)^c = A^c \cap B^c$$

Proof: $(A \cup B)^c \subset A^c \cap B^c$ and $A^c \cap B^c \subset (A \cup B)^c$

$$\begin{aligned} X \in (A \cup B)^c \\ \rightarrow X \notin A \cup B \\ \rightarrow X \notin A \text{ and } X \notin B \\ \rightarrow X \in A^c \text{ and } X \in B^c \\ \rightarrow X \in A^c \cap B^c \end{aligned}$$

$$\begin{aligned} X \in A^c \cap B^c \\ \rightarrow X \in A^c \text{ and } X \in B^c \\ \rightarrow X \notin A \text{ and } X \notin B \\ \rightarrow X \notin A \cup B \\ \rightarrow X \in (A \cup B)^c \end{aligned}$$

22

Applying Set Theory to Probability

23

Experiment

What is an Experiment?

- Method for finding some facts/conclusions

Give an example?

- "Harry Potter and the prisoner of Azkaban", is it fun?
- Stand in front of the theatre
- Ask audiences, fun or not?

Composition of an experiment

- Procedure
- Observation

Why experiment is needed?

- Uncertainty

24

Experiment

Concern about movie “Harry Potter” experiment

- Should I ask man, women, or teenager?
- Experience of the audiences
- Knowledge of the audiences

Complicated experiment → need Model

- Real experiments: too complicate
- Capture only the important parts
- Model Example:
 - Treat all audiences the same
 - Answer will only be like/dislike

25

Experiment

Same Procedure but different Observations
→ Different Experiments

Example:

1. Flip a coin 3 times, Observe the sequence of heads/tails
2. Flip a coin 3 times, Observe # of heads

26

Definition in Probability

- **Outcome**
 - Any possible observation
- **Sample Space**
 - *Finest-grain*: each outcome is different
 - *Mutually exclusive*: if one outcome occurs, other will not occur
 - *Collectively exhaustive*: every outcome must be in the sample space
- **Event**
 - Set of outcomes (Must know all outcomes)
 - Event \subset Sample Space

27

Event Examples

For an experiment:
Roll a dice, observe the shown numbers

Outcomes:

number = 1,2,3,4,5,6

Sample space:

$S = \{1,2,3,\dots,6\}$

Event examples:

$E_1 = \{\text{number} < 3\} = \{1,2\}$

$E_2 = \{\text{number is odd}\} = \{1,3,5\}$

28

Set VS. Probability

Set Algebra	Probability
Set	Event
Universal set	Sample space
Element	Outcome

29

Probability ... So easy?

The followings are not easy:

- From practical problem, model the experiment carefully
- Derive the correct sample space

30

Probability of Event, $P[\odot]$

$P[\odot]$
is a function that maps event
in the sample space to real number

From experiment: Roll a dice

Outcomes:

number = 1,2,3,4,5,6

Sample space:

$S = \{1,2,3,\dots,6\}$

Event examples:

$E_1 = \{\text{number} < 3\} = \{1,2\}$

$E_2 = \{\text{number is odd}\} = \{1,3,5\}$

$$P[E_1] = 2/6 = 1/3$$

$$P[E_2] = 3/6 = 1/2$$

31

Probability Axioms

Axiom 1: For any event A, $P[A] \geq 0$

Axiom 2: $P[S] = 1$

Axiom 3: For events A_1, A_2, \dots, A_n of mutually exclusive events
 $P[A_1 \cup A_2 \cup \dots \cup A_n] = P[A_1] + P[A_2] + \dots + P[A_n]$

32

Example Theorems

- Theorem:** If A and B are disjoint, then

$$P[A \cup B] = P[A] + P[B]$$

- Theorem:** If $B = B_1 \cup B_2 \cup \dots \cup B_n$ and $B_i \cap B_j = \emptyset$ for $i \neq j$, then

$$P[B] = \sum_{i=1}^n P[B_i]$$

33

Equally Likely

Theorem:

For an experiment with sample space $S = \{s_1, \dots, s_n\}$

if each outcome is **equally likely**,

$$P[s_i] = 1/n \quad 1 \leq i \leq n$$

34

Consequences of Axioms

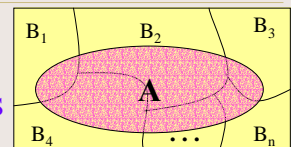
Theorem:

- $P[\emptyset] = 0$
- $P[A^c] = 1 - P[A]$
- For any A and B (not necessary disjoint)
 $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
- If $A \subset B$, then $P[A] \leq P[B]$

35

A Useful Theorem

Let B_1, B_2, \dots, B_n be mutually exclusive events whose union equals sample space S
 \rightarrow partition of S



For any event A

$$A = A \cap S = A \cap (B_1 \cup B_2 \cup \dots \cup B_n)$$

$$P[A] = P[A \cap B_1] + P[A \cap B_2] + \dots + P[A \cap B_n]$$

Theorem: $P[A] = \sum_{i=1}^n P[A \cap B_i]$

36

Conditional Probability

- If we know $P[A]$ before an experiment
 - $P[A] \approx 1$
 - $P[A] \approx 0$
 - $P[A] \approx \frac{1}{2}$
- $P[A]$ is a **priori** probability of A

37

Conditional Probability

- In practice, it maybe impossible to find the precise outcome of an experiment. Rather the outcome s_i itself.
- Example: Taking entrance exam twice
- We know that the outcome is in set B → Event B has occurred (B has many outcomes)

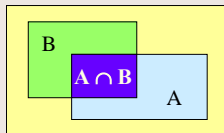
38

Conditional Probability

- **Notation:** $P[A|B]$
 - “Probability of A given B”
 - The condition probability of the event A given the occurrence of the event B

- **Definition:**

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$



- **Example:**

- Tiger Woods hits Hole-in-one

39